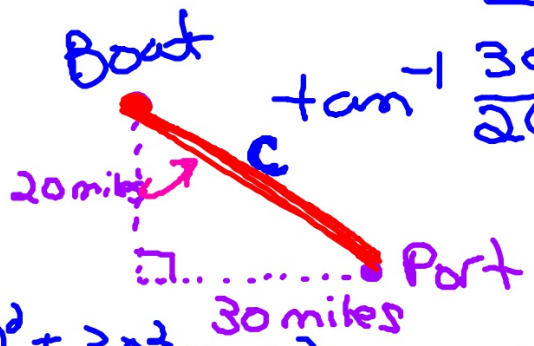
P.R.E.P.S.

A ship is 30 miles west and 20 miles north of port. The captain wants to sail directly back to port. What bearing should be taken and how far is it?

$\boxed{S\ 56.3^\circ E}$

$\tan^{-1}\frac{30}{20} = 56.3^\circ$



$20^2 + 30^2 = c^2$

$400 + 900 = c^2$

$1300 = c^2$

$c = \sqrt{1300} \approx 36$ miles

Chapter 5: Analytic Trigonometry

Introduction

In Chapter 4, you studied the basic definitions, properties, graphs, and applications of the individual trigonometric functions. In this chapter, you will learn how to use the fundamental identities to do the following:

1. Evaluate trigonometric functions.
2. Simplify trigonometric expressions.
3. Develop additional trigonometric identities.
4. Solve trigonometric equations.

Lesson 5.1A: Using Fundamental Identities

Objective: In this lesson you will learn how to recognize and write the fundamental identities and use the fundamental identities to evaluate functions.

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Using the Fundamental Identities

One common application of trigonometric identities is to use given values of trigonometric functions to evaluate other trigonometric functions.

neg. or Q II

Example 1: Use the values of $\sec u = -3/2$ and $\tan u > 0$ to find all six trigonometric functions.

Q III

$\sin u = \frac{-\sqrt{5}}{3}$ $\csc u = \frac{-3}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$

$\cos u = \frac{-2}{3}$ $\sec u = \frac{-3}{2}$

$\tan u = \frac{\sqrt{5}}{2}$ $\cot u = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2 u + \left(-\frac{2}{3}\right)^2 = 1$

$\sin^2 u + \frac{4}{9} = \frac{9}{9}$

$\sin^2 u = \frac{5}{9}$

$\sin u = \pm\frac{\sqrt{5}}{3}$

Example 2: Use the values sin x = 1/2 and cos x > 0 to find the values of all six trigonometric functions. QI

$\sin x = \frac{1}{2}$ $\csc x = 2$

$\cos x = \frac{\sqrt{3}}{2}$ $\sec x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan x = \frac{\sqrt{3}}{3}$ $\cot x = \sqrt{3}$

$\sin^2 x + \cos^2 x = 1$

$\frac{1}{4} + \cos^2 x = \frac{4}{4}$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\sqrt{3}}{2}$

$$\frac{\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \boxed{\frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Simplifying a Trigonometric Expression

Example 3: Simplify $\sin x \cos^2 x - \sin x$

$$\sin x\left(\boxed{\cos^2 x - 1}\right)$$

$$\boxed{\begin{aligned} &\sin^2 x + \cos^2 x = 1 \\ &\cos^2 x = 1 - \sin^2 x \\ &\cos^2 x - 1 = -\sin^2 x \end{aligned}}$$

$$\sin x\left(-\sin^2 x\right)$$

$$-\sin^3 x$$

Example 4: Simplify $\cos^2 x \csc x - \csc x$

$$\csc x(\cos^2 x - 1)$$

$$\csc x(-\sin^2 x)$$

$$\left(\frac{1}{\sin x}\right)(-\sin^2 x)$$

$$\frac{-\sin^2 x}{\sin x} = -\sin x$$

Homework:

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All 25-36,